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# Piecewise linear aeroelastic rotor-tower models for efficient wind turbine simulations

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**Abstract.** The development of software and methodologies able to efficiently and accurately predict the aeroelastic performance of wind turbines at low computational cost — enabling the rapid turnaround of novel concepts — is key to further reducing the levelised cost of energy. However, even the conventional low fidelity methods used for preliminary blade and rotor design are computationally expensive. The authors propose to improve the computational efficiency of dynamic aeroelastic time domain simulations by substituting conventional models with a piecewise linear aeroelastic rotor-tower model. This novel piecewise linear model is described and compared against standard modelling tools in this paper. Results suggest that piecewise linear rotor-tower models can be used to accurately capture the aeroelastic dynamic of wind turbine blades, with a significant ( $5 - 20\times$ ) reduction in computational costs.

## 1. Introduction

The global installed wind energy capacity, with the help of political, public, and economic support, is experiencing a sustained growth, with more than 50 gigawatts installed annually over the past 4 years [1]. In a global and competitive energy market, it is nonetheless important to continue improving the cost-effectiveness of wind turbines, often referred to as the levelised cost of energy (LCOE). Further reducing the time to market of novel concepts, and rapid turnaround of design ideas is also critical for industries. However, comprehensive certification-like analyses required to evaluate the LCOE of wind turbines are computationally demanding, i.e. typically many thousands of design load cases [2, 3] taking multiple days to run on a single computer core. Developing rapid design analyses and modelling tools for wind turbines is, consequently, paramount to the wind energy community. Fast software will reduce time to markets by speeding up numerical analyses. Additionally, rapid analyses tools will enable designers to undertake currently computational prohibitive tasks such as detailed monolithic aero-servo-elastic optimisation, and uncertainty quantification, both, in the long term, promising solution for further reduction of LCOE.

Numerous simulations are necessary to capture the influence of the wind's probabilistic nature on the dynamic response of a wind turbine. Most of the computational effort required to evaluate the performance of a wind turbine design is therefore spent on aero-servo-elastic time domain simulations. The focus, herein, is on the aero-servo-elastic analysis of the rotor-tower assembly,

often used to guide the design of other components through ultimate and fatigue load envelopes. Most approaches proposed in the literature [4–7] include an aerodynamic model (e.g. blade element momentum theory) coupled to structural models for the blades and tower (e.g. beam, shell), a generator model, and a controller. Although these preliminary aeroelastic simulations of wind turbines are low-fidelity, they remain computationally costly. Because, explicit time marching algorithms, typically used to solve the whole or parts of the aero-servo-elastic dynamic problem, require small time steps to ensure the solution remains stable (e.g. around 0.01 s for large wind turbines).

In the present work, the authors propose to employ a piecewise linear model to evaluate the aeroelastic performance of wind turbines, including predictions of ultimate and fatigue loads, whilst significantly reducing computational costs. More precisely, the authors investigate the following research questions.

Can a piecewise linear aeroelastic model for a wind turbine rotor-tower assembly

- closely approximate the aeroelastic dynamic response obtained with conventional models?
- capture ultimate and fatigue loads within 5-10% of conventional models?
- reduce the computational effort of DLCs 1.1 when compared to conventional models?

In this study, instead of conventional linearisation tools employed for stability analysis around the wind turbine equilibrium points [4, 8], single-point linearised models are combined into a piecewise model for aero-servo-elastic time domain simulations. This approach features two main advantages. First, the piecewise linear model is solved at each time step with an unconditionally stable implicit scheme, hence larger time steps (e.g. 0.1 s) can be used without any risk of divergence. The time step size is, therefore, solely chosen based on the frequency of the physical phenomena to be simulated. Second, the piecewise model is able to capture dynamic variations in operating conditions such as change in local wind speeds due to highly turbulent wind field, and control switching around rated wind speed. In other words, no assumption is made about the incoming flow field, or about the range of disturbances around which the model remains valid.

## 2. Methodology

The procedure used to generate the piecewise, aero-servo-elastic rotor-tower assembly, model is illustrated in Figure 1. This procedure captures the dynamic response of the rotor-tower model at different operating points and generates a piecewise linear model able to simulate the wind turbine aeroelastic dynamic response under time varying operating conditions. First, the wind turbine operating points, or equilibrium path, are estimated using a quasi-steady aeroelastic solver. Second, dynamic aero-servo-elastic simulations are performed along the estimated quasi-steady equilibrium path, each simulation runs until it converges to the dynamic equilibrium points. Although the notion of dynamic equilibrium may appear counter-intuitive, the dynamic equilibrium refers to the average operating point around which the aero-servo-elastic model operates under a constant or wind-sheared dynamic simulation. Next, the aeroelastic blade model is linearised around multiple points along the dynamic equilibrium path. The linearisation method implemented is semi-analytical, i.e. combining analytical derivations with numerical differences, and focuses on a single blade. Numerical gradients for the aeroelastic linearisation are obtained by disturbing the wind turbine model from its equilibrium during dynamic simulations. The linearised blade model is then duplicated for each blade and assembled with tower and controller models. The process is repeated for each equilibrium point to construct the piecewise linear model.

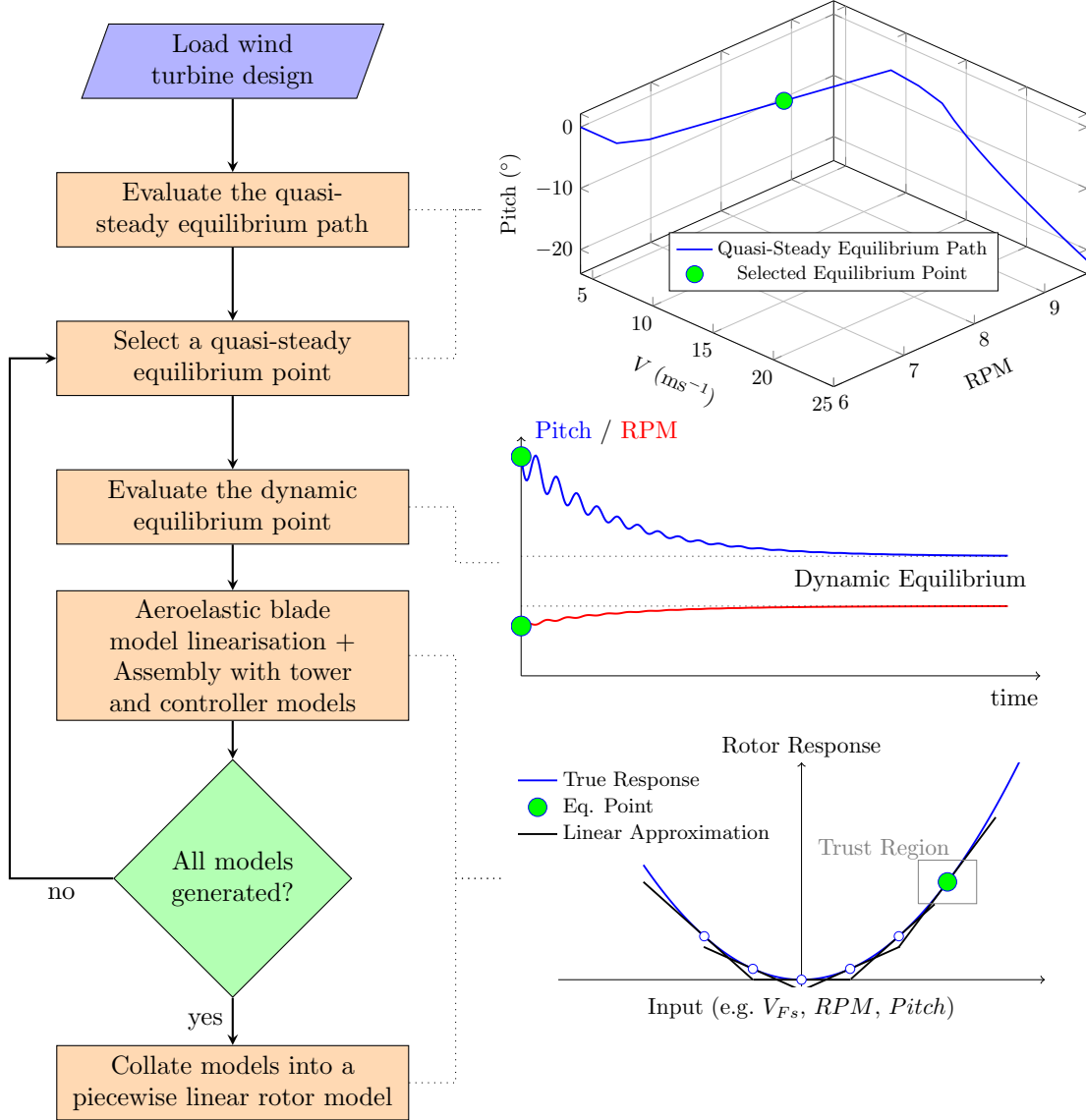


Figure 1: Procedure for generating the piecewise aero-servo-elastic rotor-tower assembly model

The in-house **Aeroelastic Turbine Optimisation Methods (ATOM)** software, developed at the University of Bristol [6, 9], is used as the baseline aero-servo-elastic solver in this work. It includes an unsteady blade element momentum theory, a dynamic stall model [10], a dynamic wake model [14], the turbulent wind field generator TurbSim [11], an in-house PID controller, a cross-section modeller, and a suite of structural beam models (modal, linear [12], and non-linear [13]). ATOM is employed to determine the dynamic equilibrium points around which the linearised model are generated, with the help of numerical differentiation. The aero-servo-elastic linearised rotor-tower model is represented in a typical state space form

$$\begin{aligned} \mathbf{x} &= \mathbf{x}_{\text{eq}} + \Delta \mathbf{x} \\ \mathbf{y} &= \mathbf{y}_{\text{eq}} + \Delta \mathbf{y} \end{aligned} \quad (1)$$

$$\begin{aligned} \Delta \dot{\mathbf{x}} &= [\mathbf{A}]\Delta \mathbf{x} + [\mathbf{B}]\Delta \mathbf{U} + [\mathbf{D}_x]\Delta \mathbf{V}_{\text{Fs}} \\ \Delta \mathbf{y} &= [\mathbf{C}]\Delta \mathbf{x} + [\mathbf{D}_y]\Delta \mathbf{V}_{\text{Fs}} \end{aligned} \quad (2)$$

where  $\Delta \mathbf{x}$  and  $\Delta \mathbf{y}$  describe linear variations of state and output vectors from their equilibrium values, respectively  $\mathbf{x}_{\text{eq}}$  and  $\mathbf{y}_{\text{eq}}$ . The primary system disturbance,  $\Delta V_{\text{Fs}}$ , denotes local variations in freestream wind velocities along the blade span. The state space matrix and the state vector expand as

$$[A]\Delta \mathbf{x} = \begin{bmatrix} \begin{bmatrix} A_A & 0 & 0 \\ 0 & A_A & 0 \\ 0 & 0 & A_A \end{bmatrix} & A_{CA} & \begin{bmatrix} A_{SA} & 0 & 0 \\ 0 & A_{SA} & 0 \\ 0 & 0 & A_{SA} \end{bmatrix} \\ A_{AC} & A_C & A_{SC} \\ \begin{bmatrix} A_{AS} & 0 & 0 \\ 0 & A_{AS} & 0 \\ 0 & 0 & A_{AS} \end{bmatrix} & A_{CS} & \begin{bmatrix} A_S & 0 & 0 \\ 0 & A_S & 0 \\ 0 & 0 & A_S \end{bmatrix} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x}_{A,b1} \\ \Delta \mathbf{x}_{A,b2} \\ \Delta \mathbf{x}_{A,b3} \\ \Delta \mathbf{x}_C \\ \Delta \mathbf{x}_{S,b1} \\ \Delta \mathbf{x}_{S,b2} \\ \Delta \mathbf{x}_{S,b3} \end{bmatrix}, \quad (3)$$

where  $\Delta \mathbf{x}_{A,bi}$  and  $\Delta \mathbf{x}_{S,bi}$  are the aerodynamic and structural states of blade  $i$ , respectively. The remaining control states, including collective pitch and angular velocity are included in  $\Delta \mathbf{x}_C$ . Note that the three blades aerodynamic  $[A_A]$  and structural  $[A_S]$  models are equivalent. Furthermore, there is currently no direct interactions between blades, i.e. blades solely interact through the hub/tower connection. The aerodynamic-structural coupling, including aerodynamic damping, is modelled through matrices  $[A_{SA}]$  and  $[A_{AS}]$ . Last, but not least, the controller model  $[A_C]$  and its interactions with the aerodynamic and structural states are encompassed in the remaining matrices,  $[A_{CA}]$ ,  $[A_{AC}]$ ,  $[A_{CS}]$ , and  $[A_{SC}]$ .

The control matrix and vector

$$[B]\Delta \mathbf{U} = \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ U_{\text{Tgen}} \\ U_{\text{Colpitch}} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (4)$$

only account for collective pitch, and generator torque control for variable speed operations. The influence of a change in free wind speeds from the equilibrium condition, i.e.  $\Delta \mathbf{V}_{\text{Fs}}$ , on the

state variables are captured by the disturbance matrix  $[D_x]$  given as

$$[D_x]\Delta V_{Fs} = \begin{bmatrix} \left[ \frac{\partial \mathbf{x}_{A,b1}}{\partial V_{Fs,b1}} \right] & 0 & 0 \\ 0 & \left[ \frac{\partial \mathbf{x}_{A,b2}}{\partial V_{Fs,b2}} \right] & 0 \\ 0 & 0 & \left[ \frac{\partial \mathbf{x}_{A,b3}}{\partial V_{Fs,b3}} \right] \\ \left[ \frac{\partial \mathbf{x}_C}{\partial V_{Fs,b1}} \right] & \left[ \frac{\partial \mathbf{x}_C}{\partial V_{Fs,b2}} \right] & \left[ \frac{\partial \mathbf{x}_C}{\partial V_{Fs,b3}} \right] \\ \left[ \frac{\partial \mathbf{x}_{S,b1}}{\partial V_{Fs,b1}} \right] & 0 & 0 \\ 0 & \left[ \frac{\partial \mathbf{x}_{S,b2}}{\partial V_{Fs,b2}} \right] & 0 \\ 0 & 0 & \left[ \frac{\partial \mathbf{x}_{S,b3}}{\partial V_{Fs,b3}} \right] \end{bmatrix} \begin{bmatrix} \left[ \begin{array}{c} \Delta V_{Fs,b1,s1} \\ \Delta V_{Fs,b1,s2} \\ \dots \\ \Delta V_{Fs,b1,sN} \end{array} \right] \\ \left[ \begin{array}{c} \Delta V_{Fs,b2,s1} \\ \Delta V_{Fs,b2,s2} \\ \dots \\ \Delta V_{Fs,b2,sN} \end{array} \right] \\ \left[ \begin{array}{c} \Delta V_{Fs,b3,s1} \\ \Delta V_{Fs,b3,s2} \\ \dots \\ \Delta V_{Fs,b3,sN} \end{array} \right] \end{bmatrix}, \quad (5)$$

where  $\Delta V_{Fs,bi,sj}$  refers to the change in free stream velocity at the aerodynamic segment  $j$ , located along the blade  $i$ . In its current implementation, a linear structural blade model is used during linearisation, meaning that non-linear geometrical deflections are not captured by the current linearised model. Whilst this is a known limitation of the on-going work, the piecewise linearisation method proposed can be extended to include nonlinear geometrical effects. Once the piecewise rotor-tower aeroelastic model is generated, dynamic simulations of design load cases (e.g. DLC 1.1, 1.3) are run. The process is illustrated in Figure 2. Two features are noteworthy. First, an implicit scheme is used to solve the aeroelastic problem at each time step. Hence, aeroelastic dynamic simulations are guaranteed to remain stable, provided the system itself is stable. That is a significant advantage as it allows using relatively large time step and is consequently faster than explicit solvers. Second, time varying effects such as change in control dynamics (e.g. switching operating region), or change in operating conditions (e.g. wind direction, magnitude) can be captured by tracking the dynamic equilibrium path and updating the rotor-tower model as its operating point changes.

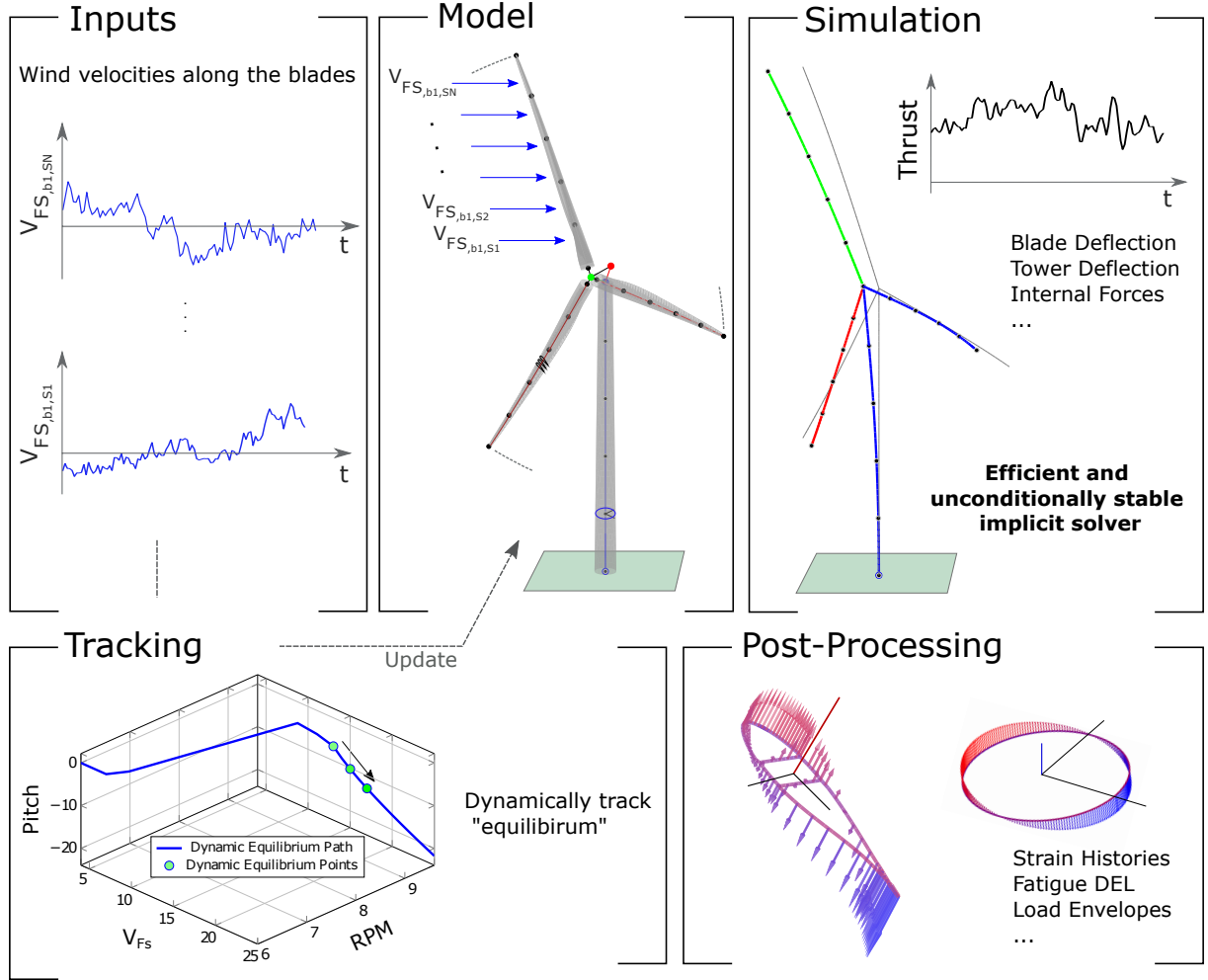


Figure 2: Dynamic simulations using the piecewise linear aeroelastic model

### 3. Results

In this section, aeroelastic predictions of the proposed piecewise linear model are compared against conventional models. Although the control strategy can significantly alter the rotor-tower behaviour, we limit our study to aeroelastic simulations during which the operating point can change but the controller remain inactive. This approach is justifiable as the authors are primarily interested in evaluating the accuracy of the piecewise model in predicting aeroelastic loads.

The DTU 10 MW wind turbine model is employed due to its freely available geometrical and structural data [15]. Two computational expensive design load cases are considered, normal power production (DLC 1.1) and extreme turbulence model (DLC 1.3). In both cases, the piecewise model predictions are compared against *ATOM*. We start by comparing time domain aerodynamic and structural outputs between the piecewise linear (PL) model and *ATOM*, for a single turbulent DLC 1.1 simulation with a mean wind speed of  $15 \text{ ms}^{-1}$ . The parameters compared in Figure 3 include, the angle of attack, lift and drag coefficients, internal reaction forces, blade displacements and twist, root bending moments (RBM), the rotor thrust, and tower top fore-aft displacements. Figure 3 demonstrates that the piecewise aeroelastic model can accurately capture the main dynamics responses of the DTU 10 MW wind turbine blades and tower, including local and global aerodynamic and structural parameters.

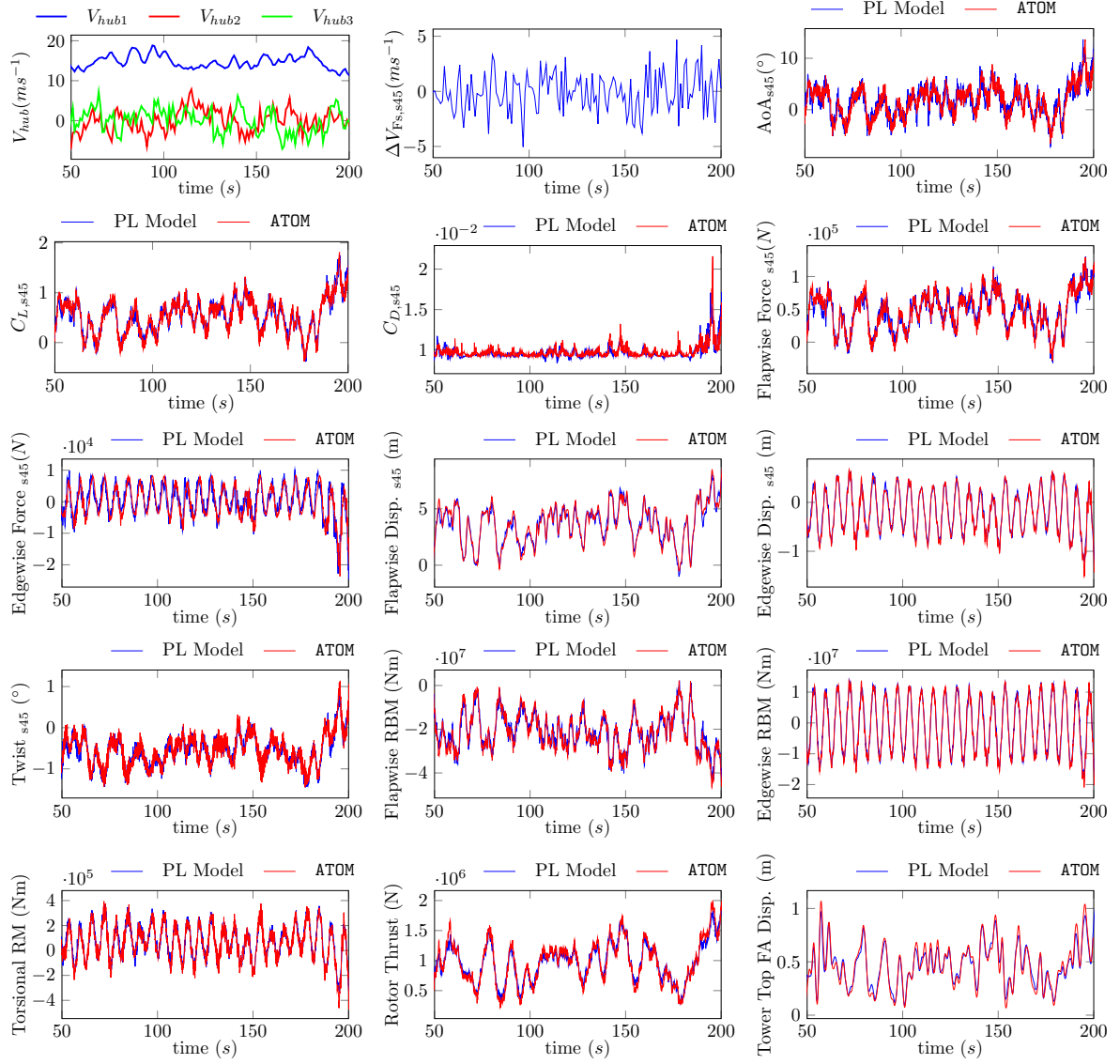


Figure 3: Comparison between the piecewise linear (PL) model and conventional wind turbine analysis with ATOM

Next, the statistical distributions of loads and displacements obtained with the piecewise linear model are compared against ATOM. Results are shown in Figures 4 and 5. Additionally, comparisons between extrema and damage equivalent loads (DEL) are shown in Table 1. One can observe that the statistical distributions for loads and displacements are, in general, well-approximated by the piecewise linear model. Maximum discrepancies are observed at the distribution tails, highlighted in red in Table 1, which are well-known to be extremely sensitive to modelling assumptions. It is therefore critically important to use a robust statistical extrapolation scheme, in order to determine extreme design driving loads. Nonetheless, many of the extrema compared are within the 10% of the expected results, with maximum discrepancies observed for edgewise, torsion and thrust loads.



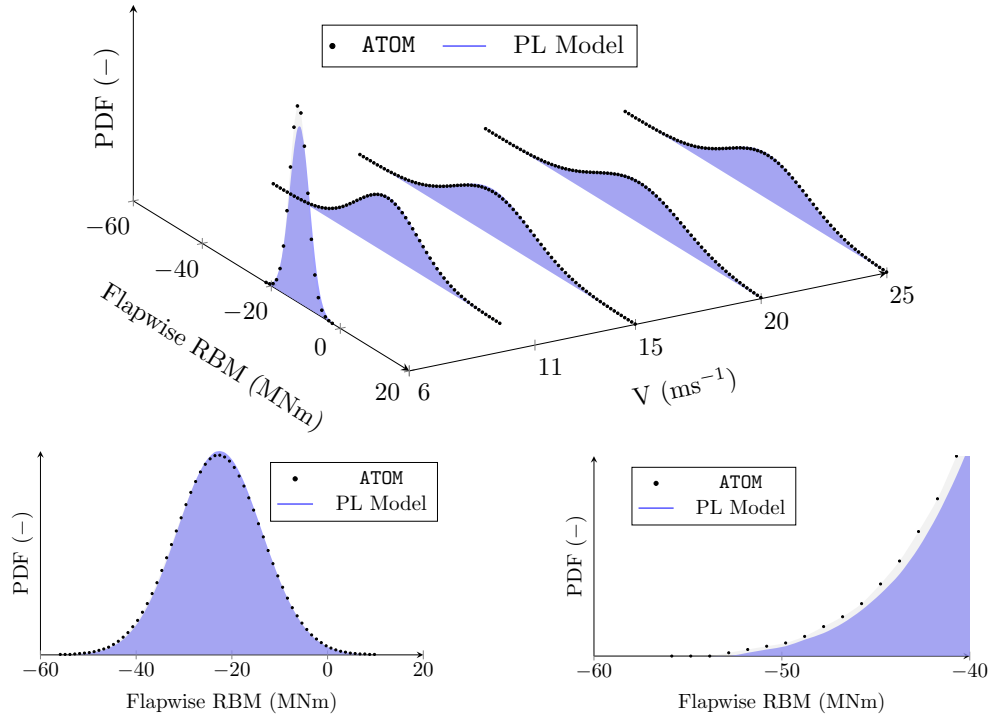


Figure 4: Statistical Analysis of Flapwise RBM, DLC1.1 and 1.3, with zoom in  $11 \text{ ms}^{-1}$

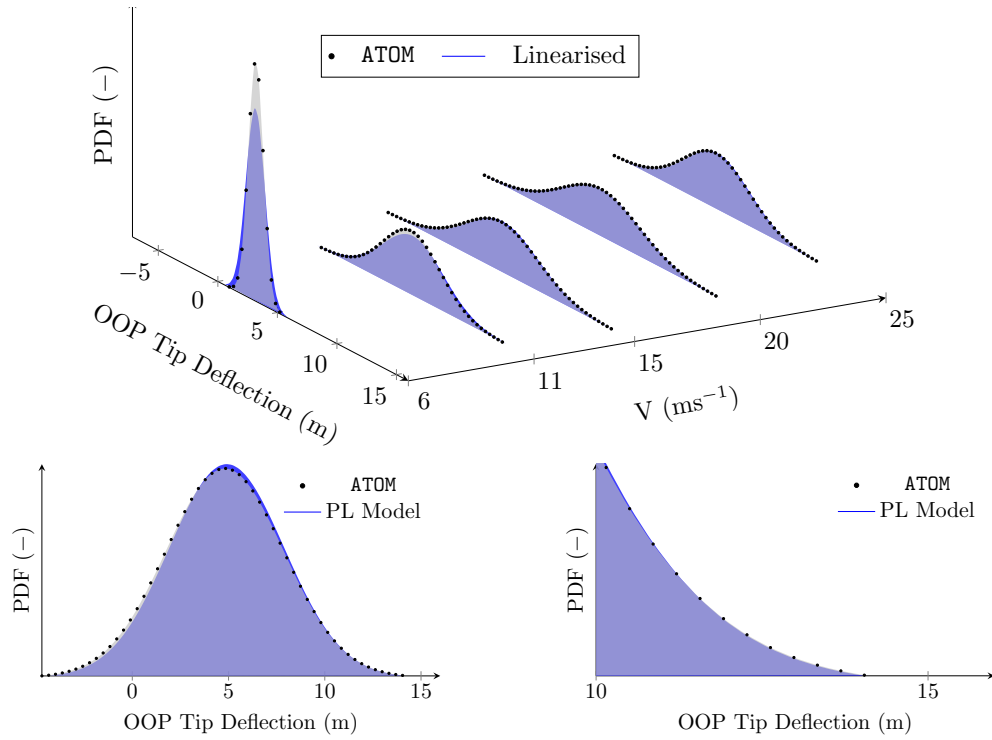


Figure 5: Statistical Analysis of Out-of-Plane (OOP) blade tip deflection, DLC1.1 and 1.3, with zoom in  $15 \text{ ms}^{-1}$

Table 1: Results Summary

Description	PL Model	ATOM	Error (%)
Max. Flapwise Root Force (MN)	0.497	0.545	8.819
Max. Edgewise Root Force (MN)	0.915	1.030	10.76
Max. Torsional Root Moment (MN.m)	1.470	1.510	2.140
Max. Edgewise Root Bending Moment (MN.m)	22.70	22.60	-0.419
Max. Flapwise Root Bending Moment (MN.m)	23.40	24.60	4.886
Min. Flapwise Root Force (MN)	-1.090	-1.070	-2.101
Min. Edgewise Root Force (MN)	-0.592	-0.629	5.813
Min. Torsional Root Moment (MN.m)	-1.910	-2.170	12.111
Min. Edgewise Root Bending Moment (MN.m)	-35.10	-39.10	10.082
Min. Flapwise Root Bending Moment (MN.m)	-61.30	-59.70	-2.770
DEL: Blade Root Flapwise Moment (MN.m)	42.90	46.50	7.680
DEL: Blade Root Edgewise Moment (MN.m)	37.60	39.60	5.076
DEL: Blade Root Torsional Moment (MN.m)	2.28	2.47	7.652
DEL: Tower Top Thrust Force (MN)	1.89	2.12	10.980
DEL: Tower Top FA Moment (MN.m)	34.00	35.10	3.179
DEL: Tower Top SS Moment (MN.m)	34.00	39.60	14.200
DEL: Tower Top Yaw Moment (MN.m)	32.30	34.00	4.885
Max. Flapwise Tip Displacement (m)	15.93	15.11	-5.470
Max. Edgewise Tip Displacement (m)	4.508	4.746	5.0195
Tower Clearance Constraint (-) (must be $> 0$ )	5.753	5.785	0.551

### Computational cost

The PL approach is significantly faster than the conventional BEM method. Although accurately evaluating computational efforts can be difficult, the average run times for a 600s-simulation under normal operating conditions (DLC 1.1), carried on a desktop PC, are showed in Table 2. Results are averaged from 36 simulations, including 12 mean wind speeds selected from  $5 - 25 \text{ ms}^{-1}$ , and considering 3-turbulent seeds per wind speeds. The reported speed-up is explained as follows. First, the BEM convergence loop is replaced by computationally efficient linear operations. Second, larger time-marching steps can be used due to the PL solver unconditionally stable implicit scheme. The results presented in Table 2 only consider DLC 1.1. Although it is possible to extend the PL approach to many more design load cases, the overall computational gains from these, already fast, BEM runs may not provide the same incentive.

Table 2: Average run times for a DLC 1.1 600s-simulation, averaged from 36 simulations.  
Run on a desktop PC (quad core i7 3.40GHz).

Method	Timestep (s)	Real time (s)	Approximate speed-up
BEM	0.02	490	-
PL Model	0.02	110	$4.5\times$
PL Model	0.1	30	$16\times$

#### 4. Discussion and Concluding Remarks

This paper has presented the application of a novel piecewise linear aeroelastic model of wind turbine rotor-tower assembly for the rapid analysis of wind turbine design load cases. The authors have compared the accuracy of the piecewise linear aeroelastic model against conventional modelling approaches, considering DLCs 1.1 and 1.3. Results indicate that the proposed piecewise model is able to accurately capture many of the wind turbine rotor-tower dynamic responses, both locally and globally. The statistical distributions of load and displacements were also shown to be well-approximated by the piecewise model. Although further confirmation is needed, the authors have identified localised non-linearities (e.g. parabolic drag coefficient, lift coefficient near stall), as important sources of discrepancy between the PL and BEM methods. In theory, discrepancies caused by localised non-linearities could be reduced/mitigated by increasing the resolution of the piecewise linear models around the quasi-steady equilibrium path, and/or by including higher order terms.

The computational speed-up provided by the piecewise linear approach are significant (e.g.  $5 - 20\times$ ), since the linearised models can readily be solved with an unconditionally stable implicit solver. As a result, time steps used for simulation can be drastically increased, when compared to explicit solver. From the authors' experiences, the time step size can be increased by an order of magnitude for large wind turbines, i.e. from  $0.005 - 0.01s$  to  $0.05 - 0.1s$ . Overall, the results are encouraging and indicate that it is possible to use piecewise linear rotor-tower models for the preliminary design of wind turbines, evaluating driving loads and displacements.

#### 5. Future Work

Further works will aim to identify and reduce the sources of discrepancy between conventional BEM and the PL model. Follow on work will also include the development of robust statistical methods for extreme load calculations, as well as including the piecewise linear model into a multi-body-dynamics framework in order to capture non-linear geometrical effect on dynamic performances. In future, the authors aim to refine the piecewise linear model and to use it for computationally expensive analyses such as holistic optimisation and uncertainty quantification.

#### 6. Acknowledgments

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